THE NET NORMAL FORCE PER CROSSING POINT: A UNIFIED CONCEPT FOR THE LOW CONSISTENCY REFINING OF PULP SUSPENSIONS

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ABSTRACT

The objectives of this article are:

– First, to theoretically propose a unified concept: the net normal force per crossing point,
– Second, to experimentally undertake refining trials on a pilot disc refiner in order to compare all concepts for the refining intensity and to validate the chosen one.

We will begin by re-visiting the old concepts of the refining intensity, in the low consistency regime. After a theoretical proof based upon the physics of the phenomena, applied to beaters and industrial refiners, a unified concept of the refining intensity is proposed and strengthened: the net normal force per crossing point.

Then, experimentations are undertaken on a pilot refiner (single disc) in hydracyle (or batch) conditions. More precisely, the effects of the grinding codes and of the average crossing angle...
of the bars are analyzed in a set of 6 refining trials. For these experimentations, different engineering concepts of the refining intensity are compared (specific edge load $B_e$, specific surface load $SS_L$, modified edge load $MEL$, net tangential force per crossing point and net normal force per crossing point). These refining intensities should allow to analysing the cutting kinetics of fibres.

All the chosen engineering concepts reach this goal more or less however the net normal force per crossing point is the best tool. Indeed, through the range of the data concerned, it revealed a clear monotonous evolution with the cutting kinetics on fibres. The more is the net normal force per crossing point, the more is the cutting effect on fibres.

1 THEORETICAL APPROACH

1.1 Case of a beater

1.1.1 Parallel bars

In the earlier 1887, Jagenberg [1] studied the case of a beater where the bars were parallel to each other and to the axis of the roll as it can be seen on figure 1. He realized that the bar contact area could have a role in quantifying the refining action on fibres. The bar contact area $A_c$ is the real area (m$^2$) obtained when the bars of the roll overlap the bars located on the bedplate.

In order to calculate this engineering parameter $A_c$, we must begin as Jagenberg did by defining the variables appearing in figure 1. $D_R$ is the diameter of the roll (m) and $a_R$, $a_S$ are, respectively the width (m) of bars on the roll, named rotor, and the width of bars on the bedplate, named stator. $l_0$ is the width (m) of the bedplate; in this typical case of parallel bars beater, the width of the bedplate is the same as the common length of the bars (m) on the bedplate or stator bars. $n_R$ and $n_S$ are, respectively the number of bars on the roll, and on the bedplate.

Jagenberg performed the fraction of the developed area of the roll covered by the roll bars at diameter $D_R$, and then this term was multiplied by the area covered by the stator bars on the bedplate. This calculation gives the bar contact area $A_c$:

$$A_c = \frac{n_R a_R l_0}{\pi D_R} n_S a_S l_0$$  \hspace{1cm} (1)

Another expression can be obtained after simplification of the term $l_0$ in the fraction referring to roll parameters.

$$A_c = \frac{n_R a_R n_S a_S l_0}{\pi D_R}$$  \hspace{1cm} (2)

Jagenberg defined another engineering parameter as the cumulative edge length $L_c$ since this term appears clearly in equation (2):

$$L_c = n_R n_S l_0$$  \hspace{1cm} (3)

Hence, he pointed out two engineering parameters:

- the cumulative edge length $L_c$ (m);
- the bar contact area $A_c$ during the overlapping of the roll in front of the bedplate.

However, he did not write the relation between these two engineering variables $L_c$ and $A_c$ for calculating the cumulative edge length $L_c$ when the bar contact area $A_c$ is known.

By introducing equation (3) in the bar contact area given by equation (2), the relation can be expressed as follows:

$$L_c = \frac{\pi D_R A_c}{a_R a_S}$$  \hspace{1cm} (4)

![Figure 1. Geometry of a beater.](image)
At this point of our development, it is interesting to find the expressions of \( n_R \) the number of bars on the roll and \( n_S \) the number of bars on the bedplate versus the beater variables as the bar width, \( a_R, a_S \) (m), the bar length \( l_R, l_S \) (m) and the length of the bedplate \( L \) (m).

If we complete our description of the beater technology, other variables must be introduced as the width of grooves (m), both on the rotor \( b_R \) and on the stator \( b_S \), see figure 1.

\[
\begin{align*}
    n_R &= \frac{\pi D_R}{a_R + b_R} \\
    n_S &= \frac{L_S}{a_S + b_S}
\end{align*}
\]

(5)

If we replace these expressions of the bar numbers (5) in the equation (2) of the bar contact area \( A_c \), we obtain:

\[
A_c = \left[ L_R l_0 \right] \frac{a_R a_S}{(a_R + b_R)(a_S + b_S)}
\]

(6)

This new development leads to define the bar contact area \( A_c \) from the global overlapping area given by the term between brackets. Hence, a new fraction appears and we will see in further developments that this fraction is also obtained in case of the other refining technologies as disc or conical refiners. This fraction only includes the bar width of the rotor \( a_R \) and that of the stator \( a_S \) and the groove width of the rotor \( b_R \) and that of the stator \( b_S \).

1.1.2 Inclined bars

In 1906 and 1907, Kirchner [2] and Pfarr [3] gave an accurate calculation of the bar contact area in case of inclined bars, both on the roll (rotor) with an angle \( \alpha_R \) and on the bedplate (stator) with an angle \( \alpha_S \). Kirchner [2] hence demonstrated that the bar contact area \( A_c \) was independent of the two angular parameters of bars \( a_R \) and \( a_S \), he obtained the equation (7) identical to equation (6) given for the case of parallel bars.

Pfarr [3] looked after an expression of the bar contact area similar to that previously given by Jagenberg in equation (2). He first calculated the bar numbers in the case of inclined bars both for the rotor and for the stator and obtained:

\[
\begin{align*}
    n_R &= \frac{\pi D_R \cos \alpha_R}{a_R + b_R} \\
    n_S &= \frac{L_S \cos \alpha_S}{a_S + b_S}
\end{align*}
\]

(8)

Then, by introducing these bar numbers in the Kirchner's equation (7), Pfarr [3] obtained the expression of the bar contact area for the case of inclined bar beaters:

\[
A_c = \left[ L_R l_0 \right] \frac{a_R a_S \sin \theta}{\cos \alpha_R \cos \alpha_S \pi D_R}
\]

(9)

We can observe the similarity between equations (2) and (9). The last one is only an extension of the previous written by Jagenberg for the case of parallel bars beater. This can be seen by replacing the angular parameters by \( \theta \).

Is it possible to obtain, similarly, an extension of the cumulative edge length \( L_c \) in case of inclined bars? To answer to this question, we will use equation (4). Then, by replacing \( A_c \), given by equation (9) in the linear relation (4), it comes without any ambiguity:

\[
L_c = \frac{\pi D_R}{a_R a_S \cos \alpha_R \cos \alpha_S \pi D_R} \frac{n_R a_R n_S a_S l_0}{a_R a_S \cos \alpha_R \cos \alpha_S \pi D_R}
\]

(10)
After simplification, we can obtain the expression of the cumulative edge length in case of beaters with inclined bars.

\[ L_\text{c} = \frac{n_R n_S l_0}{\cos \alpha_R \cos \alpha_S} \]  

(11)

If we replace the angular parameters by 0°, the previous expression already given by Jagenberg is obtained.

The cumulative edge length given by equation (11) yields three more observations.

Firstly, this cumulative edge length is found to be independent of geometrical angular parameters since the bar contact area is independent of these parameters according to Kirchner.

Secondly, the cumulative edge length is an engineering parameter previously defined [4], as the reference edge length \( L_{\text{REF}} \) (m). Hence, it is the same quantity as the cumulative edge length.

Thirdly, if \( l_0 \), the width of the bedplate is also the bar length in case of parallel bars, it is not the bar length in case of inclined bars. The real length of inclined bars (different for the rotor and for the stator) seems to appear in a complex manner as follows:

\[ L_\text{c} = n_R n_S \frac{l_0 - l_0}{\cos \alpha_R \cos \alpha_S} \]  

(12)

This is probably one reason why there were some misunderstandings in the past when different authors tried to extend the cumulative edge length from beaters to disc or conical refiners. (We will use at least equation (11) to perform the calculation of the cumulative edge length for beaters, disc and conical refiners.) So Roux has used \( L_\text{c} \) also for disc refiners which is not right.

When \( \alpha_R \) and \( \alpha_S \) are different from zero angles (case of inclined bars), during the relative motion of the roll in front of the bedplate, a certain number of crossing zones are generated. Kirchner [2] named these zones “crossing points” and decided to calculate their number \( N_{CP} \). He gave the expression of the area of a single bar crossing (m²) when the bar angle is known:

\[ N_{CP} \quad \text{crossing points (Kirchner named it)} \]  

\[ A_{CP} = \frac{a_R a_S}{\sin (\alpha_R + \alpha_S)} \]  

(13)

Then, the calculation of the number of crossing points \( N_{CP} \) was obtained by the ratio of the bars contact area \( A_c \) by equation (9) by the area of a single bar crossing as shown:

\[ N_{CP} = \frac{A_c}{A_{CP}} \]

Crossing point \( A_{CP} \) by equation (13). The expression of \( N_{CP} \) is also given in Baker’s book [5] in chapter 2.

\[ N_{CP} = \frac{n_R n_S l_0}{\pi D R} \left( \tan \alpha_R + \tan \alpha_S \right) \]

(14)

Again, we developed another relationship between the number of crossing points \( N_{CP} \) and the cumulative edge length \( L_\text{c} \) with the same assumptions concerning the angles. It is possible to obtain:

\[ N_{CP} = \frac{L_c}{\pi D R} \sin (\alpha_R + \alpha_S) \]

(15)

1.1.3 Inclined bars: net normal force per crossing point

(Roux and his co-workers [4] introduced the concept of net normal force (N) and refining intensity (N) for a beater. They calculated the net normal force by the following formula which includes the global friction coefficient \( f \))

\[ \frac{F_{\text{net}}}{P_{\text{net}}} = \frac{F_{\text{shear}}}{P_{\text{shear}}} = \frac{P_{\text{net}}}{D R \alpha} \]  

(16)

In equation (16), \( P_{\text{net}} \) is the net power consumed by the refiner and \( N \) is the rotation speed of the rotor. Now, dividing the net normal force by the number of crossing points \( N_{CP} \), as given by equations (14) or (15), it is possible to quantify the net normal force per crossing point.

From equation (14), we obtain the following expression of the net normal force per crossing point given by equation (17):

\[ N_{CP} = n_R n_S l_0 \left( \tan \alpha_R + \tan \alpha_S \right) N \]

(17)

The expression between brackets in the denominator was empirically introduced by Meltzer [6], as the “extended edge length.” He considered “the projected bar length as the efficiency of a bar to cross counterbars.” Our theoretical approach naturally introduces the concept of extended edge length already proposed by Meltzer and then adds to its legitimacy.

The net normal force per crossing point is our candidate for describing the refining intensity in the low consistency refining process of pulp suspensions.

If the equation (15) is chosen to quantify the number of crossing points
that the extension of the cumulative edge length for the beaters in case of inclined bars was given by equation (11). This last one differs from equation (19) where the average bar length \( \bar{L} \) is not clearly defined.

We propose to clarify the concept of the cumulative edge length, in the case of a disc refiner, through a physical analogy between a slice of the beater and an elementary annulus of disc.

On figure 3, the pulp suspension motion in the beater occurs in the plane formed by two unit vectors \( \vec{t}_p \), \( \vec{t}_s \), perpendicular to the roll axis. Now, if we consider a slice of the beater in the direction of the unit vector \( \vec{t}_p \), it means that the width of the bedplate is also the width of the slice beater \( dl_0 \). The determination of the elementary cumulative edge length \( dL_c \) for this slice of the beater is given by equation (11) written in a differential form as follows:

\[
dL_c = \frac{\eta_R \eta_S \cdot dl_0}{\cos \alpha_R \cos \alpha_S}
\]

(21)

For a beater, the bar numbers are independent of the slice of the beater as mentioned previously. So, the integration of equation (21) is easy to perform with this type of technology and leads to equation (11) already mentioned through a global understanding at the beginning of this article.

![Figure 3. Slice of a beater.](image)

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\[
F_{net}^{net} = \frac{P_{net}}{N_{CP}} \\
= \frac{P_{net}}{f \cdot \sin(\alpha_R + \alpha_S)}
\]

(18)

It is noted that the numerator is simply the Specific Edge Load, \( B_s \) (J/m or N) defined lately by Brecht and Siewert [8].

However, as the cumulative edge length and the reference edge length are the same physical quantity, the specific edge load is also the same quantity as the reference specific edge load, already introduced in [4]. Again, this adds legitimacy to the concept of the reference specific edge load that is exactly the same as the specific edge load.

However, we must admit that the specific edge load is not sufficient to adequately describe the refining intensity since two adding parameters are influencing variables in equation (18):
- the global friction coefficient \( f \) of the pulp versus bar materials;
- the sinus of the bar crossing angle given by the algebraic sum \( \alpha_R + \alpha_S \).

1.2 Disc refiner analysis

1.2.1 The cumulative edge length

The extrapolation of the cumulative edge length was proposed in 1958 by Wulfsch and Flucher [7]. They introduced the following well known expression [5], to calculate the cumulative edge length per revolution (m):

\[
L_{eq} = \frac{n_R \cdot n_S \cdot L}{1 + \frac{n_R}{n_S} \cdot \bar{L}}
\]

(19)

A new variable \( \bar{L} \) was introduced: the average bar length (m). Their understanding was based upon a proposed extension of Jagenberg's equation (3) as stated in Baker's book [5].

Later, Brecht and Siewert [8] used the expression (19) to calculate the Specific Edge Load \( B_s \) (J/m) by equation (20), following the initial ideas of Wulfsch and Flucher:

\[
B_s = \frac{P_{net}}{L \cdot N}
\]

(20)

However, our understanding of the cumulative edge length for disc refiners is somewhat different. At first glance, we already demonstrated in this article
On figure 4, we drew an elementary annulus comprised between the radius \( \rho \) and the radius \( \rho + d\rho \). The pulp suspension motion is supposed to be circular with this type of disc refiner in the direction of the unit vector \( \mathbf{T}^* \), for each radius, analogous to the case of a beater.

However, the major difficulty with a disc refiner is given by the different situations that occur with time during the relative motion of the rotor disc in front of the stator disc.

In that case, how the bar angles for both the rotor and the stator, may be defined? If one expression must be used for extrapolation, it should be that given by equation (21), which is certainly not yet currently used in the industrial practice.

A careful consideration of the disc geometry, as it was done for example in [9], can lead to the local determination of the numbers of bars on the rotor \( n_R \) and on the stator \( n_S \) as function of the local radius \( \rho \) with the corresponding angles \( \Phi_R \) and \( \Phi_S \) for the bar inclinations for the rotor and the stator, respectively:

\[
\begin{align*}
 n_R(\rho) &= \frac{2\pi \rho \cos(\Phi_R)}{a_R + b_R} \\
 n_S(\rho) &= \frac{2\pi \rho \cos(\Phi_S)}{a_S + b_S}
\end{align*}
\]  

(22)

An analogous form of equation (21), between a slice of the beater and an annulus of a disc refiner leads to the calculation of the elementary cumulative edge length, by substituting \( d\rho \) by \( d\rho \).

\[
dL(U, \rho, \chi) = \frac{n_R(\rho)n_S(\rho)d\rho}{\cos \Phi_R \cos \Phi_S}
\]  

(23)

Hence, in order to find the expression of the cumulative edge length for a disc refiner, integration must be performed over the refining annulus, limited by the internal radius \( \rho_i \) (m) and the external radius \( \rho_e \) (m), using equations (22) and (23).

\[
L_c = \int_{\rho_i}^{\rho_e} \frac{n_R(\rho)n_S(\rho)d\rho}{\cos \Phi_R \cos \Phi_S} = \frac{4\pi^2}{3(a_R + b_R)(a_S + b_S)} \rho_e^3 - \rho_i^3
\]

(24)

Some observations can be given. Equation (24) reveals that \( L_c \) is independent of the bar angles on the rotor and on the stator. It confirms the result already obtained by Kirchner when beaters with inclined bars were considered, see equation (11).

The cumulative edge length is the same physical quantity as the reference edge length already introduced in [9], since it appeared naturally in the calculations. However, in this article, it is demonstrated through a physical analogy between beaters and disc refiners. The correct extrapolation of equation (11) for beaters to industrial disc refiners and their complex geometry is given by equation (24).

If we follow the equation (19) for the practical calculation of the cutting edge length, we should evaluate this quantity by the following:

\[
CEL = \frac{\rho}{n_Rk} S_k \Delta \rho_k
\]  

(25)

In this expression, the disc corona is divided in \( p \) elementary annulus where
the determination of the bar numbers on the rotor \( n_{R_k} \), on the stator \( n_{S_k} \) is performed for each annulus. All these numbers are then combined in order to give the cutting edge length (CEL) according to equation (25).

Due to differences in case of inclined bars between the practical determination of the CEL given by the formula (25) and real measurements, the Stock Preparation TAPPI Committee decided to include some cosine terms in the expression (25). The next expression (26) is given for example, in the Technical Information Papers of the TAPPI Association for the determination of the cutting edge length (CEL):

\[
CEL = \sqrt{\sum_{k=1}^{p} n_{R_k}^2 \Delta \phi_k / \cos(\phi_R)} \sqrt{\sum_{k=1}^{p} n_{S_k}^2 \Delta \phi_k / \cos(\phi_S)} \quad (26)
\]

In this last equation, average angles for the rotor and the stator are empirically introduced, which could give an expression “close to” our theoretical solution.

However, we suggest using the proposed equation (24) which has some theoretical justification instead of the empirical expression, given by equation (26). Furthermore, in the TAPPI expression, both set variables are gathered angular and non-angular parameters.

In the proposed equation (24), no reference is made to angular parameters, so we need to complete our physical analysis to “naturally” introduce these angular parameters, this will be done in the next paragraph.

1.2.2 The net normal force per crossing point

Equation (16) gives the net normal force versus the net power consumed for a beater; this expression includes a tangential velocity in the denominator. If one wants to develop a similar tangential velocity equation for a disc refiner, we must find its equivalent diameter or radius.

This can be done by calculating, over the disc annulus, the average tangential velocity defined as follows [10]:

\[
2.\pi. < \rho > . N = \frac{2.\pi. p. N. 2\pi. \rho. d\rho}{\pi. (\rho_s^2 - \rho_l^2)}
\]

(27)

Hence, the average radius \(< \rho >\) (m) is given by the expression:

\[
< \rho > = \frac{2.\pi. p. N. 2\pi. \rho. d\rho}{\pi. (\rho_s^2 - \rho_l^2)}
\]

(28)

Dividing the net normal force by the number of crossing points requires the determination of this last number. Due to complex geometrical situations occurring in the rotating motion of the rotor in front of the stator, it is only an average calculation of this number which is performed.

The average number of crossing points \( N_{CP} \) was already calculated by Roux [9] and its expression reduced to a simplified one in the case of a small sector angle:

\[
N_{CP} = \frac{\pi (\rho_s^2 - \rho_l^2)}{(a_R + b_R)(a_S + b_S)} \sin(\phi_R + \phi_S)
\]

(29)

Now, the last step is the determination of the net normal force per crossing point, calculated on the average, for a disc refiner, by introducing the average radius – equation (28) and the average number of crossing points – equation (29), we finally obtain:

\[
F_{n, net} = \frac{P_{net}}{f.\pi. 2. < \rho > . N. N_{CP}} = \frac{P_{net}/(L_e . N)}{f.\sin(\phi_R + \phi_S)}
\]

(30)

Again, some observations can be drawn. The numerator of the net normal force per crossing point is the reference specific edge load or the specific edge load since these variables express the same physical quantity: the net energy per unit length of bar edge (J/m) or (N).

This quantity: the numerator does not contain any angular parameter. These last variables are included in the denominator. In the previous TAPPI expression (26), all the variables were gathered but, in fact, there are decoupled in equation (30).

The physical analogy obtained between equations (18) and (30) is the clear demonstration that the refining intensity chosen as the net normal force per crossing point is a unified concept.

It can be applied to different technologies as a beater or an industrial disc refiner with different geometries.

The case of a conical refiner, not treated here, may be treated exactly in the same way.
2 INTERPRETATIONS OF THE INDUSTRIAL PARAMETERS

2.1 Description of the refining installation

The refining trials were undertaken on our Laboratory pilot installation [10, 11], as described on figure 5. The pilot is equipped with a single disc refiner (SD) running in hydracycle mode.

![Figure 5. Laboratory refining installation.](image)

It means that the refining energy is distributed to the pulp suspension in cycles. The refiner is an automated process with gap clearance sensor and axial force sensor. This specific metrology was previously described in [12] and will not be detailed in this article. It allows us to measuring the normal and tangential forces and to deducing both the global friction coefficient.

In order to define the different engineering parameters which will be modified during the set of refining trials, we must first remind the geometrical configuration of a disc plate, as shown on figure 6.

In this typical design, a given sector is composed of parallel bars in an annulus comprised of an internal radius \( r_i \) and an external radius \( r_e \). On this drawing, the sector has an angular periodicity of \( \delta \) per revolution meaning that the sector angle is equal to \( \theta = 60^\circ \). Usually, the sector angle is common for the rotor and the stator. On figure 6, the angle of the first bar, located at the right side, is named the “grinding angle” and written by convention: \( \alpha \) for the rotor disc and \( \beta \) for the stator disc. The understanding of the angular parameters is the same for the rotor or the stator in a first glance.

Depending on the bar chosen for the rotor disc, the bar inclination angle is varying from \( \alpha \) to \( \alpha + \theta \). Hence, a typical value of the bar inclination angle, for each sector (rotor or stator) is given by the average value, we then obtain the following expressions:

\[
\begin{align*}
\phi_R &= \alpha + \frac{\theta}{2} \\
\phi_S &= \beta + \frac{\theta}{2}
\end{align*}
\]

(31)

If a rotor plate is superimposed in front of a stator plate, by considering the average value for each plate, the angle of the bar crossings is defined as follows:

\[
\gamma = \overline{\phi_R} + \overline{\phi_S}
\]

(32)

On the one hand, the quantification of the average angle for the entire bar crossings is obtained by replacing the terms of equations (31) in equation (32):

\[
\overline{\gamma} = \alpha + \beta + \theta
\]

(33)

On the other hand, the number of bars located at a given radius \( r \), can be analytically calculated, by introducing the previous angles \( \phi_R \) and \( \phi_S \) in the following expressions given for the rotor, then for the stator:

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2.2 Description of the refining conditions

The pulp suspension tested in these investigations was a market pulp of non bleached Kraft Canadian softwood. Neither the characterization of the pulp nor the essence was undertaken since our aim in this article was based upon an assessment of engineering parameters.

During the six refining trials, the dry mass of the pulp in the tank was the same at $m = 14$ kg and the solid consistency chosen at $C = 3.5\%$ (or 35 kg/m$^3$). A constant volumetric flow measured by the flow meter was kept at $Q = 10$ m$^3$/h. Hence, the time period of a cycle was constant at $V/Q = 2.4$ min (or 144 s).

The refining trials were undertaken on the same single disc refiner with the following engineering parameters: internal radius $\rho_i = 0.065$ m; external radius $\rho_e = 0.150$ m. These values allow the calculations of the ratio $k$ of the internal to the external radius of the annulus:

$$\frac{k_\rho}{\rho_e} = 0.433$$

and that of the average radius $<\rho>$, considering the equation (28) rewritten by using the previous ratio:

$$<\rho> = \frac{2(1-k^2)}{2(1-k^2)} \rho_e = 11.31 \times 10^{-2} m$$

The net power was kept constant during one refining trial. The rotation speed of the refiner was also taken at a constant value for all the refining trials: $N = 1500$ rev/min or 25 rev/s.

For the disc plates, made of stainless steel, the groove depth was constant at 6.4 mm, corresponding to a grinding code of either (2-2-4) or (3-3-4) expressed classically in sixteenth of inches. We remind to our reader that the grinding code is classically a set of 3 numerical values $(a-b-c)$, respectively:

\[
\begin{align*}
\eta_{Re} &= \frac{2\pi \rho_e \cos(\Phi_R)}{a_R + b_R} \\
\eta_{St} &= \frac{2\pi \rho_e \cos(\Phi_S)}{a_S + b_S}
\end{align*}
\]

(34)

2.3 Description of variable engineering parameters

The engineering parameters chosen for these investigations were:

- the common value of the bar width and groove width: $a = b$;
- the net power constant during one given trial $P_{net}$;
- the sum of the grinding angle of the rotor and the stator: $a + \beta$.

In table 1, we have gathered these different numerical values, completed by the average crossing angle $\bar{\gamma}$, given by equation (32):

<table>
<thead>
<tr>
<th>Trial no</th>
<th>$a = b$ (mm)</th>
<th>$P_{net}$ (kW)</th>
<th>$a + \beta$</th>
<th>$\bar{\gamma} = a + \beta + \theta$</th>
<th>$\bar{\varphi}_R$ (°)</th>
<th>$\bar{\varphi}_S$ (°)</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>3.2</td>
<td>16.6</td>
<td>0</td>
<td>22.5</td>
<td>16.25</td>
<td>6.25</td>
</tr>
<tr>
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<td>3.2</td>
<td>9.6</td>
<td>25</td>
<td>22.5</td>
<td>16.25</td>
<td>6.25</td>
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<td>12</td>
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<td>47.5</td>
<td>21.25</td>
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<td>21.25</td>
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<tr>
<td>5</td>
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<td>12.4</td>
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<td>16.25</td>
<td>6.25</td>
</tr>
<tr>
<td>6</td>
<td>4.8</td>
<td>24</td>
<td>0</td>
<td>22.5</td>
<td>16.25</td>
<td>6.25</td>
</tr>
</tbody>
</table>

The sum of the grinding angles $a + \beta$ was chosen for two different numerical values:

- a so-called "cutting" geometry where $a = +5^\circ$ and $\beta = -5^\circ$;
- a so-called "fibrillating" geometry where $a = +10^\circ$ and $\beta = +15^\circ$.

As it was previously mentioned in [9], the refining intensities defined by different authors in the paper literature are always proportional to the net power applied. The bar width $a$ and the average angle $\bar{\gamma}$ of the bar crossings are also known to be influential variables for characterizing the refining effects on fibres.
2.4 Determination of different refining intensities

For all the refining trials specified by table 1, we propose to calculate the different refining intensities in the following order: the specific edge load \([8]\), the specific edge load recommended according to the TAPPI engineering rules, the specific surface load \([13]\), the modified edge load \([6]\), the net tangential force per crossing point \([9]\), the net normal force per crossing point \([9]\).

2.4.1 Determination of the specific edge load

If we combine equations (20) and (24) with the correct quantification of the cumulative edge length, the determination of the specific edge load \(B_s\) is then given by the following expressions:

\[
B_s = \frac{P_{net}}{Lc \cdot N} = \frac{3(a+b)^2}{4 \pi^2 (\rho_e^2 - \rho_i^2) \cdot N} \sum_{k=1}^{p} \frac{n_{Rk} \cdot A \rho_{k}}{\cos(\Phi_k)}
\]

(37)

Two different bar widths (or groove widths) and six different net powers lead to consider a refining intensity in a range of 1 to 5.6.

2.4.2 Determination of the specific edge load according to TAPPI rules

As it is proved in this article, no angular parameter can be found in equation (37). TAPPI rules proposed to empirically introduce the angular parameters as described in the following developments:

\[
\begin{align*}
B_{s,TAPPI} &= \frac{P_{net}}{CEL \cdot N} \\
&= \frac{\sum_{k=1}^{p} n_{Rk} \cdot A \rho_{k}}{\cos(\Phi_k)} \cdot \frac{\sum_{k=1}^{p} n_{S} \cdot \rho_{S}}{\cos(\Phi_S)}
\end{align*}
\]

(38)

According to these rules, the calculation of the specific edge load can be done after the primary determination of the edge length both for the rotor \(EL_R\) and the stator \(EL_S\).

The edge length for the rotor and the stator are the respective expressions under square root signs in equations (38). For sake of simplification, we consider that the bar width \(a\), respectively the groove width \(b\), are the same for the rotor and the stator. In equations (38), the disc corona is divided in \(p\)

The net normal force per crossing point

elementary annulus. The edge length for the rotor \(EL_R\) can be determined with the help of equation (34) that counts the bar number on a given annulus:

\[
EL_R = \frac{\sum_{k=1}^{p} n_{Rk} \cdot A \rho_{k}}{\cos(\Phi_k)} = \frac{\sum_{k=1}^{p} 4 \pi^2 \cdot \rho_e^2 \cdot \cos^3(\Phi_k) \cdot A \rho_{k}}{(a+b)^2}
\]

(39)

The calculation of the edge length for the stator \(EL_S\) is formally the same as that for the rotor. If a continuous expression is preferred rather than a discrete one, the cutting edge length \(CEL\) is then given by the following equation after integration over the global corona:

\[
CEL = \frac{4 \pi^2 \cdot (\rho_e^2 - \rho_i^2)^3}{3(a+b)^2} \sqrt{\cos(\Phi_R) \cdot \cos(\Phi_S)}
\]

(40)

This last expression allows the final determination of the specific edge load according to TAPPI rules:

\[
B_{s,TAPPI} = \frac{B_s}{\sqrt{\cos(\Phi_R) \cdot \cos(\Phi_S)}}
\]

(41)

This expression empirically introduces the angular parameters that combine grinding angles and the sector angle as previously given by equations (31).

2.4.3 Determination of the specific surface load

 Lumiainen \([13]\) claimed that the bar width and the average crossing angle\( \gamma\) must be taken into account for a more precise quantification of the refining intensity. He also gave the general formula to calculate the specific surface load:

\[
SSL = \frac{B_s}{aR + aS} \cdot 2 \cdot \cos(\gamma/2)
\]

(42)

In case of the same bar width for the rotor and the stator, the formula is simplified to the following expression:

\[
SSL = \frac{B_s}{a} \cdot \cos(\gamma/2)
\]

(43)
2.4.4 Determination of the modified edge load

With this refining intensity given by Meltzer [6], the modified edge load includes the bar width, the groove width and the average crossing angle as it is expressed in the general case:

\[ MEL = \frac{B_0}{2 \tan(\gamma / 2)} \left( \frac{a + b}{a} \right) \]  

(44)

When the bar width and the groove width are the same, this equation can be simplified and the bar width then disappears, it only remains in the expression of the specific edge load \( B_0 \):

\[ MEL = \frac{B_0}{\tan(\gamma / 2)} \]  

(45)

2.4.5 Determination of the net tangential force per crossing point

This quantity can be obtained in two steps. Firstly, the net tangential force is deduced from the net power applied to the average tangential velocity as expressed by equation (27):

\[ F_{net} = \frac{P_{net}}{2 \pi \cdot \rho \cdot N} \]  

(46)

Secondly, the average number of crossing points is calculated precisely with the following equations already given by Roux in [9]:

\[ N_{CP} = \frac{\pi (p_s^2 - p_l^2)}{(a_R + b_R) (a_S + b_S) \cdot \sin(\gamma^*)} \]

\[ \sin(\gamma^*) = \sin(\gamma) \left( \frac{\sin(\theta / 2)}{\theta / 2} \right)^2 \]  

(47)

After replacement of the average number of crossing points in the expression of the net tangential force per crossing point, we obtain the following equation which can be practically used to calculate the required value:

2.4.6 Determination of the net normal force per crossing point

The authors of this article have theoretically demonstrated that the unified concept for quantifying the refining intensity was the net normal force per crossing point. In fact, it means that a physical term is lacking, this term is the global friction coefficient \( f \). The expression of the net normal force per crossing point hence comes:

\[ F_{net} = \frac{B_0}{N_{CP} \cdot f \cdot \sin(\gamma^*)} \]  

(49)

2.4.7 Observations on the different refining intensities

Some general observations can be drawn after calculation of the different candidates for the refining intensity. All the refining intensities, except the specific edge load, incorporate angular parameters. The specific surface load includes one more engineering parameter: the bar width on the average. The modified edge load includes the bar width and the groove width.

However, the net normal force per crossing point incorporates a new physical quantity that has never been introduced before (except by the authors themselves): the global friction coefficient.

Our last step is then to calculate the candidates for quantifying the refining intensity on fibres during the pulp refining in low consistency regime.

2.5 Calculation of different refining intensities

We have gathered all the empirical and physical quantities in table 2 in order to see if they are classified in the same order for the 6 refining trials undertaken on our pilot.

Some observations can be given. First, the star angle \( \gamma^* \) is very close to the average crossing angle \( \gamma \) as it can be seen through a comparison between the numerical values for all the six refining trials in tables 1 and 2. Second, the cutting speed \( L_c, N \) shares the refining trials in two categories: (1,2) and (3,4,5,6) whereas the average number of crossing points \( N_{CP} \) leads to three categories: (1,2), (3,4), (5,6). At this point, we can observe that the level of discrimination is better by using concepts that incorporate angular parameters.
Third, for all the refining trials undertaken, the different chosen refining intensities cover a range of 1 to 6, which is sufficiently representative for a benchmark.

Fourth, the best discriminating refining intensities among the six candidates are given by the three last columns. This result is proved when one wants to classify the trial numbers from the smallest to the highest refining intensity; we obtain for the six different candidates:

<table>
<thead>
<tr>
<th>Refining Intensity</th>
<th>Classification of the trial numbers towards increasing intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_i$</td>
<td>$2 &lt; 4 &lt; 1 &lt; 3 &lt; 5 &lt; 6$</td>
</tr>
<tr>
<td>$B_{r,Tappi}$</td>
<td>$2 &lt; 1 &lt; 4 &lt; 5 &lt; 3 &lt; 6$</td>
</tr>
<tr>
<td>SSL</td>
<td>$2 &lt; 4 &lt; 1 &lt; 5 &lt; 3 &lt; 6$</td>
</tr>
<tr>
<td>MEL</td>
<td>$4 &lt; 2 &lt; 3 &lt; 1 &lt; 5 &lt; 6$</td>
</tr>
<tr>
<td>$F_{lcr}/N_{cp}$</td>
<td>$4 &lt; 2 &lt; 3 &lt; 1 &lt; 5 &lt; 6$</td>
</tr>
<tr>
<td>$F_{n}/N_{cp}$</td>
<td>$4 &lt; 2 &lt; 3 &lt; 1 &lt; 5 &lt; 6$</td>
</tr>
</tbody>
</table>

If we remind that the specific edge load, its calculation according to TAPPI engineering rules and the specific surface load are empirical concepts, we are not surprised by the results obtained. None of this candidate leads to the same classification as the others.

So, it is clear that these tools should be used with great care for a quantification of the refining effects on fibres, especially the shortening effect on fibres.

3 PRACTICAL RESULTS

3.1 Cutting kinetics of fibres

Among the main refining effects, the cutting on the fibres is more directly related to the refining intensity. Hence, the refining intensity concept was first developed in order to quantify this shortening effect on fibres. Our purpose is to find the refining intensity which would be the best tool to quantify the cutting kinetics on fibres, among the six possible refining intensities presented in this article.

3.2 Experimental results

For one given refining trial, the average weighted fibre length $L_f$ was measured with a Morfi analyser at different times, which means at different specific
energies consumed by the pulp. The six refining trials are performed with a constant power applied $P_{ref}$ and a constant rotation speed $N$ as it was previously presented in paragraph 2.2. The following tables summarise the data obtained for the six refining trials.

### Table 4. Pulp properties versus net specific energy for refining trial n°1

<table>
<thead>
<tr>
<th>Pulp property</th>
<th>$E_{np}$ [kWh/T]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$L_t$ [mm]</td>
<td>2.158</td>
</tr>
<tr>
<td>$^\circ$SR</td>
<td>15</td>
</tr>
<tr>
<td>WRV [g/100g]</td>
<td>108</td>
</tr>
</tbody>
</table>

### Table 5. Pulp properties versus net specific energy for refining trial n°2

<table>
<thead>
<tr>
<th>Pulp property</th>
<th>$E_{np}$ [kWh/T]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$L_t$ [mm]</td>
<td>2.158</td>
</tr>
<tr>
<td>$^\circ$SR</td>
<td>15</td>
</tr>
<tr>
<td>WRV [g/100g]</td>
<td>108</td>
</tr>
</tbody>
</table>

### Table 6. Pulp properties versus net specific energy for refining trial n°3

<table>
<thead>
<tr>
<th>Pulp property</th>
<th>$E_{np}$ [kWh/T]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$L_t$ [mm]</td>
<td>2.158</td>
</tr>
<tr>
<td>$^\circ$SR</td>
<td>15</td>
</tr>
<tr>
<td>WRV [g/100g]</td>
<td>108</td>
</tr>
</tbody>
</table>

Table 7. Pulp properties versus net specific energy for refining trial n°4

<table>
<thead>
<tr>
<th>Pulp property</th>
<th>$E_{np}$ [kWh/T]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$L_t$ [mm]</td>
<td>2.158</td>
</tr>
<tr>
<td>$^\circ$SR</td>
<td>15</td>
</tr>
<tr>
<td>WRV [g/100g]</td>
<td>108</td>
</tr>
</tbody>
</table>

Table 8. Pulp properties versus net specific energy for refining trial n°5

<table>
<thead>
<tr>
<th>Pulp property</th>
<th>$E_{np}$ [kWh/T]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$L_t$ [mm]</td>
<td>2.158</td>
</tr>
<tr>
<td>$^\circ$SR</td>
<td>15</td>
</tr>
<tr>
<td>WRV [g/100g]</td>
<td>108</td>
</tr>
</tbody>
</table>

Table 9. Pulp properties versus net specific energy for refining trial n°6

<table>
<thead>
<tr>
<th>Pulp property</th>
<th>$E_{np}$ [kWh/T]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$L_t$ [mm]</td>
<td>2.158</td>
</tr>
<tr>
<td>$^\circ$SR</td>
<td>15</td>
</tr>
<tr>
<td>WRV [g/100g]</td>
<td>108</td>
</tr>
</tbody>
</table>

In this article, we will only investigate on the cutting effect on fibres and will analyse the other main effects (fibrillation and hydration) in future publications.
All results can be condensed in the same figure for an assessment of the cutting kinetics on fibres for all the experimental trials performed.
3.3 Empirical modelling of the cutting kinetics

For the range of specific energy concerned by these refining trials, a second order model fits adequately the experimental average weighted fibre length. Hence, the following equation applies with a good precision for all the refining trials performed. For the refining trial n<sup>k</sup>:

\[
L_{f-k} = a_k \left( \frac{E_{\text{net}}}{E_{\text{sp}}} \right)^2 + b_k \left( \frac{E_{\text{net}}}{E_{\text{sp}}} \right) + L_{f0}
\]

(50)

This empirical model is then used to interpolate the average weighted fibre length \( L_{f-k} \) (for the refining trial n<sup>k</sup>) at the same net specific energy consumption chosen: 150 kWh/T.

All the calculated data are summarised in Table 10.

Table 10. Interpolation of the average weighted fibre length \( L_{f-k} \) for \( E_{\text{net}} = 150 \text{ kWh/T} \) and for the six refining trials

<table>
<thead>
<tr>
<th>Trial n&lt;sup&gt;k&lt;/sup&gt;; k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{6} a_k )</td>
<td>-0.458</td>
<td>-2.546</td>
<td>-4.971</td>
<td>-3.863</td>
<td>-4.992</td>
<td>+7.979</td>
</tr>
<tr>
<td>( 10^{5} b_k )</td>
<td>-20.42</td>
<td>-6.646</td>
<td>-4.521</td>
<td>+0.142</td>
<td>-29.03</td>
<td>-69.82</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>0.9987</td>
<td>0.9932</td>
<td>0.9917</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.9999</td>
</tr>
<tr>
<td>( L_{f-t}(150 \text{kWh/T}) ) [mm]</td>
<td>1.841</td>
<td>2.001</td>
<td>1.978</td>
<td>2.073</td>
<td>1.610</td>
<td>1.290</td>
</tr>
</tbody>
</table>

The refining trial number 6 was performed with the highest refining intensity as it is proved by table 3, whatever the candidate used for defining this refining intensity. The positive sign of the coefficient \( a_k \) means a change in the curvature of the cutting kinetics on fibres, compared to the other coefficients, from \( a_1 \) to \( a_5 \), as it can be observed on figure 7.

With table 10, the six refining trials can be classified from the smallest to the highest cutting effect on fibre: 4 < 2 < 3 < 1 < 5 < 6.

Hence, in a first approach, this range obtained can be compared with the different classifications already given by table 3. The three last candidates for defining the refining intensity: the modified edge load \( MEL \), the net tangential force per crossing point \( F^\text{net}N_{\text{cf}} \), the net normal force per crossing point \( F^\text{net}N_{\text{cp}} \) succeed to evaluate the cutting effect on fibres.

3.4 Benchmark of the different refining intensities

In order to determine the level of understanding which is obtained with one given refining intensity, we will compare the different average weighted fibre length (at 150 kWh/T) for the six refining intensities.

A graphic interpretation will be privileged since it is the simplest one for performing the benchmark. In order to prepare the analysis, all the data are gathered in table 11.

We will show in the following developments the “clouds” of points between the different values of fibre length and that of refining intensity.

If we find a linear relationship between the fibre length and the corresponding refining intensity, it will mean that the refining intensity can be a good tool to quantifying the cutting effect on fibres.

Table 11. Data required for benchmarking the refining intensities

<table>
<thead>
<tr>
<th>Trial n&lt;sup&gt;i&lt;/sup&gt;</th>
<th>( L_{f-t}(150 \text{kWh/T}) ) [mm]</th>
<th>( B_t ) [J/m]</th>
<th>( B_t ) [J/m]</th>
<th>SSL [J/m]</th>
<th>( MEL ) [J/m]</th>
<th>( F^\text{net}N_{\text{cf}} ) [N]</th>
<th>( F^\text{net}N_{\text{cp}} ) [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.841</td>
<td>0.656</td>
<td>0.672</td>
<td>202.7</td>
<td>3.299</td>
<td>1.737</td>
<td>11.58</td>
</tr>
<tr>
<td>2</td>
<td>2.001</td>
<td>0.380</td>
<td>0.388</td>
<td>117.2</td>
<td>1.908</td>
<td>1.005</td>
<td>5.58</td>
</tr>
<tr>
<td>3</td>
<td>1.978</td>
<td>1.067</td>
<td>1.167</td>
<td>205.1</td>
<td>2.426</td>
<td>1.466</td>
<td>7.33</td>
</tr>
<tr>
<td>4</td>
<td>2.073</td>
<td>0.623</td>
<td>0.681</td>
<td>119.7</td>
<td>1.415</td>
<td>0.855</td>
<td>3.42</td>
</tr>
<tr>
<td>5</td>
<td>1.610</td>
<td>1.103</td>
<td>1.129</td>
<td>227.1</td>
<td>5.545</td>
<td>2.919</td>
<td>22.45</td>
</tr>
<tr>
<td>6</td>
<td>1.290</td>
<td>2.135</td>
<td>2.185</td>
<td>439.6</td>
<td>10.73</td>
<td>5.651</td>
<td>35.32</td>
</tr>
</tbody>
</table>
Hence, it could be adventurous to use in the paper industry these concepts alone to quantify the cutting effect on fibres.

**Case of the specific surface load SSL**

If the regression coefficient is better with the specific surface load than that with the two previous cases, it is also clear that the discriminating nature of this intensity is lost. At least, two pairs of fibre length cannot be discriminated with this tool as seen in figure 10.

\[ Lf_k = -0.7848E-03 . SSL + 2.1580E+00 \]
\[ R^2 = 8.1256E-01 \]

**Figure 10.** Correlation between the average weighted fibre length at \( E_{\alpha}^{\text{cut}} = 150 \text{ kWh/T} \) and the Specific Edge Load SSL.

**Case of the modified edge load MEL**

At this point, neither the specific edge load nor its corrected value according to the TAPPI engineering rules is able to account for the understanding of the cutting kinetics on fibres. This result could have been observed in table 3 where the classification of the refining intensity was not in the same order as the cutting kinetics on fibres.

\[ Lf_k = -8.4652E-02 . MEL + 2.1580E+00 \]
\[ R^2 = 9.7499E-01 \]

**Figure 11.** Correlation between the average weighted fibre length at \( E_{\alpha}^{\text{cut}} = 150 \text{ kWh/T} \) and the Modified Edge Load MEL.
In this article, we have developed a unified physical refining theory which puts the emphasis on the net normal force per crossing point. Our reader is probably now convinced that this is the best refining intensity for the interpretation of the cutting kinetics on fibres. Even if the results with the net tangential force are acceptable, the introduction of the global friction coefficient is of primary importance in understanding the cutting effect on fibres.

We also have proved that the normal forces are responsible for the cutting on fibres and that the terminology often used in the industry of “cutting angle” should be eliminated definitely, we prefer to use crossing angle, which is exactly what it is, nothing more.

This classification of the cutting effect on fibres with these data is monotonous with the proposed refining intensity that is to say the net normal force per crossing point. The more the net normal force per crossing point, the more pronounced will be the cutting effect on fibres.

The average crossing angle is not the only influencing variable since the specific edge load on one hand and the global friction coefficient on the other hand counterbalance its effect, see equation (49) that is reminded in a simplified form:

$$\frac{F_{n_{\text{net}}}}{N_{CP}} = \frac{B_s}{f \cdot \sin(\gamma)}$$

When one wants to verify if a model takes correctly into account the experimental data, the parity diagram is an interesting tool for this purpose.

A parity diagram is given on figure 14 between the estimated fibre length

![Parity diagram](image)

**Figure 14.** Parity diagram for the estimation of the average fibre length at 150 kWh/T by using the net normal force per crossing point as refining intensity.
CONCLUSIONS

In this article, we have proposed to revisit the earlier knowledge of the refining operation of pulp fibre suspension in low consistency range when it was operated on beater equipments. 

By considering the relation between the cutting edge length and the bar contact area, two fundamental variables introduced previously by Jagenberg, it was possible to theoretically justify the extrapolations:

- in a first step, from a beater with parallel bars to a beater with inclined bars;
- in a second step, from a beater with inclined bars to a disc with inclined bars also.

This last extrapolation based initially on geometrical concepts was further justified by the physical analogy that exists between a slice of the beater and an elementary annulus of disc plates.

In the literature, it is possible to find different concepts for assessing the "refining intensity". A benchmark was undertaken considering the most famous practical refining intensities as the Specific Edge Load ($B_e$), its specific value according to the TAPPI engineering rules ($B_e$-TAPPI), the Specific Surface Load ($SSL$), the Modified Edge Load ($MEL$), the net tangential force per crossing point ($F_{n1}/N_{cp}$) and the net normal force per crossing point ($F_{n2}/N_{cp}$).

Six refining trials were carried out on our refining pilot under hydracycle conditions.

Two bar widths and two average bar crossing angles were investigated. These refining trials were compared considering the shortening effect on fibres at the same net specific energy consumption of $150 \text{ kWh/T}$.

The results lead to the conclusion that three tools can be used as refining intensity to quantify the cutting effect on fibres: the modified edge load, the net tangential force per crossing point and the net normal force per crossing point. However, the net normal force per crossing point that results from our refining physical theory has the best monotonous variation with the shortening kinetics on fibres. It is graphically proved in this publication. All the other refining intensities were not sufficient in quantifying this shortening effect on fibres.

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