Mechanics and Fluid Dynamics of a Disk Refiner

A formula is proposed to express the total power consumed by a disk refiner:

\[ h_R = K_f N^2(D_0^2 - 2/3D_1^2) + K_N D_2(D_f^2 - D_0^2) + K_Q N D_1^2. \]

The first term represents the friction loss of a disk rotating in a fluid. Although this loss follows the well established laws of fluid dynamics, the magnitude of the Kf factor for conventional refiner plate designs is many times higher than that determined for other types of turbomachinery. An explanation is proposed. The second term represents the effective power applied to the stock. By applying the laws of solid rather than fluid dynamics, a theory is proposed to explain why the various formulas for inch cuts per minute, intimacy factor, etc., have not been correlated with disk refiner performance. The third term represents the power consumed in accelerating the stock to rotational velocity and is normally small enough to be disregarded. A disk design is proposed to minimize the disk friction losses and to optimize the “beating” characteristics of a disk refiner. A comparison of results made from the “proposed” and conventional designs is made.

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we decided to analyze the power consumption of a disk refiner from a purely mechanical engineering viewpoint. While bar count, bar area, angle between bars, peripheral speed, etc., have a significant effect on the papermaking qualities of the refined fibers, our concern with papermaking properties in this investigation has been limited to evaluating the results of the analysis. The purpose of this paper is to propose a mathematical model of a disk refiner in terms of an energy balance.

**ANALYSIS**

The first step in the analysis was to separate the power consumption into three principal components: the power required to revolve the disk in a fluid medium—disk friction; the power imparted to the stock—effective work; and the power required to accelerate the stock to exit velocity from the plates—pumping losses.

**Disk Friction**

One of the earliest analyses of disk friction losses was published by Schultz-Grunow (1), and simplified in (English) discussions may be found in Csanydy (2), Stepanoff (3), and Kaufmann (4). In essence, the disk friction can be expressed as a function of rotational speed to the third power and diameter to the fifth power, the well-known power number in fluid mechanics. Included in the constant is the mass of the fluid and a moment coefficient which is dependent upon the Reynolds number and relative roughness.

The principal flow pattern is outward radially along the surface of the rotating disk, and inward toward the axis along the surface of the stationary disk or housing, Fig. 1.

Our observations of disk refiners have shown that the disk friction with clear water at finite disk clearances follows the basic formula,

\[ h_P = K_D N^3 D^5 \]  

(1)

with \( K_D \) remaining essentially constant over a wide range of disk diameters, makes, and speeds, although it is much higher than that calculated by the methods of references (2), (3), or (4). However, when we consider the configuration of plates most commonly used in disk refiners—a series of bars and grooves—we realize that in effect we have a fluid coupling, and that the torque developed approaches that of some of the earlier models of fluid couplings. Since the work consumed in overcoming this torque does little except heat the water and amounts to 30–50% of total power applied, a reduction in this loss would significantly improve the efficiency of disk refiners.

A method to accomplish this improvement is proposed and demonstrated later on.

**Effective Work**

Although other methods for evaluating effective work have been proposed by Skrabak (5) and Daizelli (6), we decided to employ an analysis based upon an assumed mechanical coefficient of friction, as for example that in a disk brake clutch. A very clear derivation and explanation of the formula is given by Doughtie (7).

In a disk brake, the power dissipated is equal to a constant multiplied by the rate of revolution multiplied by the difference between the cube of the outer (OD) and the cube of the inner (ID) diameters of the contacting surfaces, if the pressure between the plates is equal across the contacting surfaces. We could then write a formula for effective horsepower as:

\[ h_{\text{P}} = K_P N (D_o^3 - D_i^3) \]  

(2)

The constant \( K_P \) includes the pressure in lb/ft², or other appropriate dimensions, and the coefficient of friction. Unfortunately, this relationship only applies when the brake lining or the refiner plates are new, and the pressure over the entire contacting surfaces is uniform.

As the brake lining or refiner plates are used, wear occurs. The wear is a function of the work performed and is proportional to the product of speed, pressure, and coefficient of friction. In order to achieve uniform wear, the work expended must be constant over the entire surface. Hence, if the coefficient of friction remains constant, the product of linear speed and pressure must be constant. Since the linear speed varies directly with diameter, the pressure must vary inversely with the diameter.

This means that at the inside periphery of the refiner plates there is high pressure and low speed, generally considered to be a "cutting" condition. At the outside periphery of the disk, there is high speed and low pressure, considered to be a hydrating or fibrillating condition.

Consequently, as soon as the refiner plates have been "broken-in," the formula must be rewritten (7):

\[ h_{\text{P}} = K_P N D_i (D_o^3 - D_i^3) \]  

(3)

The significance of this change is that as the inside diameter of the disk becomes smaller, the product of the inside diameter and the difference between the squares of the outside and inside diameters gradually increases from zero, when \( D_i = D_o \) or \( D_i / D_o = 1 \), to a peak at a ratio of \( D_i / D_o \) of approximately 0.6, and then gradually decreases at lower ratios, \( D_i / D_o \leq 0.6 \).

**Pumping Losses**

Under Disk Friction we discussed losses due to recirculation only. When we actually pass fluid through the machine, the mass must be accelerated from inlet velocity to exit velocity, and the power required is proportional to the mass (specific weight x 32.2) times the square of the velocity. Thus,

\[ h_{\text{P}} = K_P Q N^2 D_o^2 \]  

(4)

where the constant \( K_P \) includes the gravity constant, 32.2 ft/sec², and other appropriate factors as required to be consistent with the dimensions chosen for the quantity \( Q \), rate of revolution \( N \), and diameter \( D \). In the normal capacity range of disk refiners, this only amounts to 1 or 2% of total horsepower applied.

There are of course other minor losses such as bearing and packing gland friction, but they are relatively quite small and essentially constant and therefore disregarded in our analysis.

**Total Horsepower**

We may now combine the above elements into an expression representing the total power of a disk refiner with conventional plates:

\[ h_{\text{P}} = K_P N^3 D_o^5 + K_P N D_i (D_o^3 - D_i^3) + K_P Q N^2 D_o^2 \]  

(5)

where

- \( h_{\text{P}} \) = total brake horsepower applied;
- \( N \) = rate of revolution, rpm;
- \( D_o \) = outside diameter of refiner plates, ft;
- \( D_i \) = inside diameter of bar and groove circle of the refiner plates, ft;
- \( K_P \) = disk friction constant, which is

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directly proportional to the density of the fluid handled and varies with plate geometry;

\[ K_w = \text{a constant which includes the coefficient of friction between fibers and plate and fiber and fiber as well as average separating pressure between plate "contacting" surface; } \]

\[ K_f = \text{a constant which includes the gravity constant and the density of the fluid. } \]

**OPTIMIZING THE DISK REFINER**

We determined experimentally that the disk friction constant for conventional bar and groove plates is at least three times as high as the constant for disks having the same general dimensions but no bars or grooves. This leads to the conclusion that we might more properly write the formula for disk friction in a disk refiner as:

\[ h_p = K_w N(D_a - D_i) + K_f N^2 D_i \]  

where the first term represents loss in the bar and groove area, and the second term losses in an area with smooth surfaces. Obviously, the larger the area with no bars and grooves, the smaller will be the disk friction losses.

Next, if we consider the effective work as

\[ h_p = K_w N D_i (D_a - D_i) \]  

for any given size of disk refiner at constant speed \( N \), the outside diameter, \( D_a \), is constant.

If we plot effective work, Eq. (3), as a percentage of total work normally applied to a refiner, against the ratio of inside to outside diameter, \( D_i/D_a \), and do the same for disk and other losses, Eqs. (4) and (6), we obtain the relationships shown in Fig. 2.

The majority of the refiner plates in use today have a \( D_i/D_a \) ratio in the range of 0.45 to 0.50, and the manufacturer rates the horsepower capacity of the refiner (taking due consideration of the type of stock and consistency) at this plate ratio. Note that if our theory of mechanical friction is correct, the refiner should have the same effective work capacity at a \( D_i/D_a \) ratio of about 0.75, whereas it actually peaks out at about 0.6.

At a \( D_i/D_a \) ratio of 0.75, the losses are about 10% less than those at 0.45. Additionally, at the same average pressure between the plates, with a \( D_i/D_a \) of 0.45, the pressure at the inside periphery of the plates is 138% of the average, that at the outside periphery is 62%, an increase of more than 2.2 to 1. With a \( D_i/D_a \) of 0.75, the same pressures are 114 and 86%, respectively, or an increase of only 1.33 to 1.

If this concept of mechanical friction as effective work is correct, then we should be able to demonstrate this in a disk refiner.

**EXPERIMENTAL**

Our experimental work was conducted in two phases. First, we constructed a single disk refiner of clear plastic, with 20-in. OD plates of conventional design (see Figs. 3 and 4). With this equipment, and a high-speed movie camera (3000 and 9000 frames/sec), we were able to verify the flow patterns described under Disk Friction.

In the second phase of our experimental program, we ran conventional and modified plates in a commercial, 22-in. disk refiner. For refining tests, and also for disk loss tests, we removed the feeder and inner section of some standard 22-in. refiner plates, Fig. 5. The disk friction horsepower for the conventional and the rim plates was as shown in Fig. 6. We also fitted some 20-in. plates on the 22-in. refiner and ran disk friction losses with the conventional plate, with the disk plate only having the grooves filled with epoxy resin, and with the stator and rotor plates both smoothed with epoxy resin.

These results are shown in Fig. 7.

Using the conventional and the rim plates, we refined a bleached, west coast kraft at 4% consistency, feeding the refiner at a 50 ton/day rate (oven-dry basis) using 240 brake hp for the conventional plates and 205 brake hp for the rim plates. We removed inner bars from a set of available standard plates, and the \( D_i/D_a \) ratio was about 0.85, somewhat higher than optimum.

At the horsepower applied, the effective work was higher than that that should have been applied had we followed the curve of Fig. 2. The intensity of refining was therefore greater on the rim plates than on the conventional plates, and the freeness dropped more rapidly with the rim plates when plotted against total work done, Fig. 8, than with the conventional plates.

However, in spite of the increased in-

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**Fig. 2.** Effective and disk friction hp vs. \( D_i/D_a \) (see Appendix II).

**Fig. 3.** Side view of plastic refiner.

**Fig. 4.** End view of plastic refiner.
hp = \frac{Nf[D_i^3 - \frac{1}{2}D_o^3]}{D_i} + \frac{Nf[D_i^3 - D_o^3]}{D_i} + KfQN\sqrt{D_i}

This has been verified, to some extent, by experimental laboratory work with a commercial-size disk refiner.

More uniform treatment of pulp can be achieved by using refiner plates with a larger ratio of inside to outside diameter than have been normally employed, without sacrificing capacity of the refiner, and with a reduction in the total work required for a given degree of refining.

It is our hope that other investigators will explore further the effect of the \(D_i/D_o\) ratio, and thus perhaps we will achieve a better understanding of the “beating” action of a disk refiner, particularly since various plate configurations affect the physical properties of the finished paper.

**APPENDIX I. DERIVATION OF EQ. (3)**

From ref. (7), p. 344, Eqs. (14), (15)

\[
T = \pi nf_{\text{max}}r_0(r_o^2 - r_i^2)
\]

where

- \(f\) = coefficient of friction
- \(n\) = number of friction areas
- \(f_{\text{max}}\) = maximum pressure between plates, lb/in.\(^2\) (at \(r_i\))
- \(r_i\) = inside diameter of plates, in.
- \(r_o\) = outside diameter of plates, in.
- \(T\) = torque, in.-lb

Let

\[
N = \text{speed of rotation, rpm}
\]

\[
D_i = \text{inside diameter of plates, in.} = 2r_i
\]

\[
D_o = \text{outside diameter of plates, in.} = 2r_o
\]

Multiply each side of Eq. (1) by \(N\), and substitute \(D_i/2\) for \(r\)

\[
\frac{NT}{12} = \text{ft-lb/min}
\]

and

\[
\frac{NT}{550} = \text{hp} = \frac{NT}{6000}
\]

Dividing each side of Eq. (2) by 6000, and substituting hp for \(NT/6000\), gives

\[
\text{hp} = \frac{Nf_{\text{max}}}{52,800}(D_i^3 - D_o^3)
\]

For any given set of conditions, say maximum rated horsepower capacity of a refiner on a particular stock at specified consistency, the fiber-to-fiber and fiber-to-plate coefficient of friction \(f\),

**Fig. 5.** Conventional and rim plates.

**Fig. 6.** Disk friction curves, 22-in. refiner.

**Fig. 7.** Disk friction curves, 20-in. plates.

**Fig. 8.** Freeness vs. hp-day/ton, conventional and rim plates.
or the average of in and out, is constant, and the maximum pressure \(p_{max}\) between plates is constant. We may then substitute one constant, \(K_a\), for \(\pi D_{max}/52.800\):

\[h_p = K_a N D (D_o^2 - D_i^2)\]

**APPENDIX II. CALCULATIONS FOR \(h_p\) AT VARIOUS \(D_o/D_i\) RATIOS (FIG. 2)**

Under average operating conditions, disk friction and pumping and other losses amount to approximately 33% of total rated horsepower capacity of the machine. The effective horsepower is 67% of the total, or

\[h_p (\text{rated}) = 0.67 h_p (\text{total})\]

In Eq. (3), let

\[h_p (\text{rated}) = 67\% \]

\[D_o = 1\] (consider OD of machine unity)

\[D_i = 0.45\] (common ratio of ID to OD of plates)

\[K_a N = C (\text{maximum pressure and average in-out coefficient of friction remain constant})\]

Then,

\[67 = C \times 0.45 (1 - 0.45^2)\]

\[C = 187\]

Substituting,

\[h_p = 187 D_o (1 - D_i^2)\]

Calculate \(h_p\) for various values of \(D_i\), or \(D_i/D_o\):

\[\begin{array}{cccc}
D_i & D_o & (1 - D_i^2) & D_o (1 - D_i^2) h_p \\
0.45 & 0.202 & 0.798 & 0.358 & 67 \\
0.50 & 0.250 & 0.750 & 0.375 & 70 \\
0.60 & 0.360 & 0.640 & 0.383 & 72 \\
0.70 & 0.400 & 0.510 & 0.357 & 67 \\
0.80 & 0.640 & 0.360 & 0.288 & 54 \\
\end{array}\]

**LITERATURE CITED**


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